SFB 1120

Annual report 2018

Subproject B6

Title

Dreidimensionale Modellierung und effiziente numerische Beschreibung des Kontakts zwischen Festkörpern und Flüssigkeiten

Projectmanagement /-processing

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Task definition

The aim of the sub-project (TP) B6 is the development of efficient numerical methods for the thermomechanical contact between the cooling melt and a solid including the influence of imperfect mechanical contact due the surface roughness or air pockets. In order to achieve this goal, it is necessary to consider numerous aspects of the numerical analysis. In the previous year, the creeping flow solver was developed as well as the coupling schemes for fluid-structure interaction problems. To build on this, now the focus is set on the boundary element method (BEM) and its formulation thereby improving the accuracy as well the efficiency. Further development of the solver has been required to extend its capabilities to new application cases, allowing now to solve problems like linear elasticity, heat conduction and as a popular request of other sub-projects the electrostatic interaction. Further, recent advancements in the developed framework allow to deal with unsteady potential problems. To combine the new developments into one complex multiphysical system, the coupling of BEM and finite element method (FEM) is revised. In particular the influence of the external fields solved by BEM onto the thin structures like liquid and solid membranes and shells solved by FEM is considered. The formulations for the thin structures are well established in the group of contact mechanics and adhesion of AICES and are used in the framework created by this sub-project.

Procedure

In general, the BEM is an approach for solving linear partial differential equation. The main idea is to express the domain solution in term of the boundary distributions thereby reducing the dimensionality of the problem by one order. Beside this very appealing advantage, the conventional BEM comes with a central drawback: singular kernels have to be integrated numerically. To address this problem special quadrature rules has been developed. One of those quadrature rules is implemented in TP B6. Further, the non-singular BEM is investigated and implemented. These two methods together reduced for the test cases the error significantly. In the further pursue of increasing capabilities of the BEM, the Stokes formulation was extended to solve linear elastic problem, as well as the Laplace equation solver was implemented allowing to solve potential problems like heat conduction or electrostatic interaction.

Furthermore, the discretization was investigated. It was achieved by means of the isogeometric analysis (IGA) and appropriate choice of the collocation points. One of the benefits of using this kind of discretization is that the geometry is represented in the computational analysis exactly improving the accuracy of the solution.

In order to broaden the boundary element formulation to the cases of the nonhomogeneous and time-dependent problems, the dual reciprocity (DR) boundary element formulation has been implemented. This development allows to include body forces in creeping flow, to solve Poisson problems and unsteady Laplace problems.

Results

The implemented DR-BEM is investigated and validated with unsteady heat conduction problems. One example is an annual region with Neumann boundary conditions (constant heat flux) on the outer surface and Dirichlet boundary conditions (constant temperature) on the inner surface. The resulting temperature field at different time steps and the surface temperature evolution is shown in Fig.1. It can be seen that the surface temperature approaches the steady state solution.

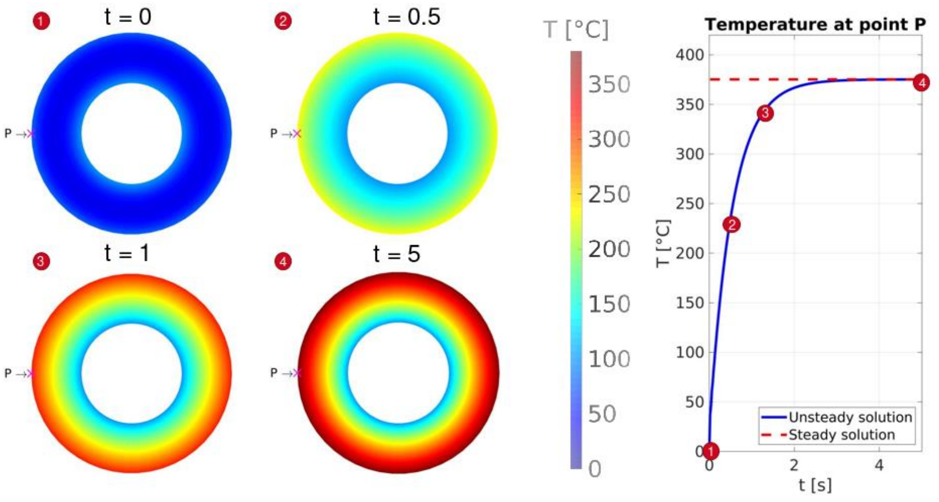

Figure 1. U*nsteady heat conduction in an annual region*

The non-singular BEM is investigated in steady Stokes flow problems. A rotating sphere in a viscous fluid is considered here. The geometry of the problem is discretized isogeometrically and the collocation point are chosen by means of the Greville abscissae. Fig.2. depicts that the non-singular BEM (bottom) results in a significantly smaller traction error than the conventional BEM (top).

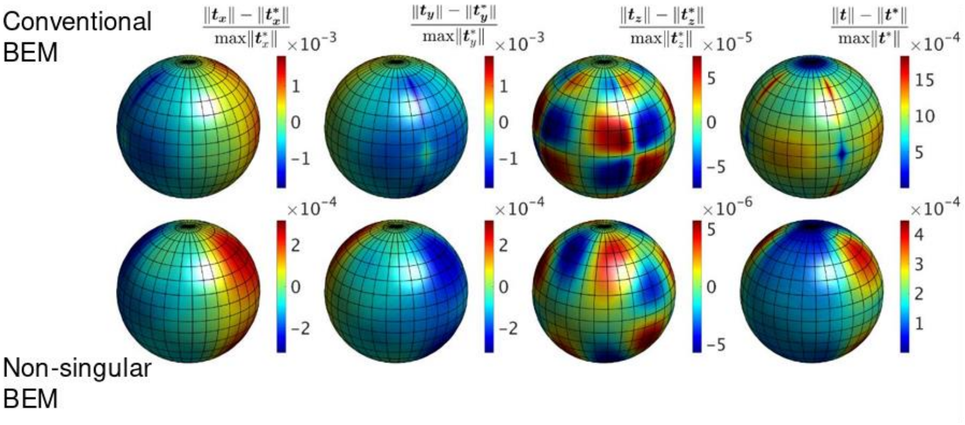

Figure 2. *Comparison of the conventional and the non-singular BEM.*

The setup of a long range electroelastic interaction problem in the free space of two conducting spheres can be seen in Fig. 3. The spheres are charged with the same potential $\acute{\varphi}^{\star}$and separated by a distance of $3R_0$. The Kirchhoff-Love shell formulation is used to model the behavior of the deformable surface. During the simulation the potential on the surfaces are gradually increased. In Fig. 4 shows the initial configuration (left) and the deformed configuration for $\bar{\varphi}^{\star} = 140\varphi_0$ (right).

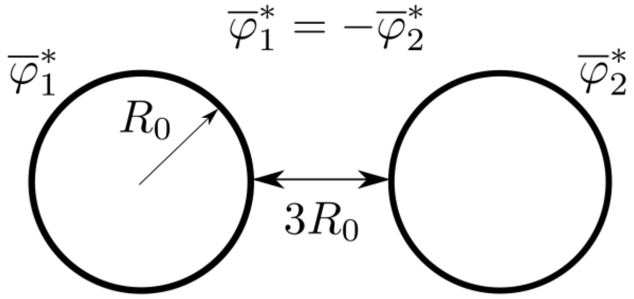

Figure 3. *Problem setup for an electroelastic interaction in free space.*

This example allows to test the BEM formulation for the potential problems as well as developed coupling scheme. Additional challenge in this example was the discretization of the deforming spheres with IGA. Repeated control points on the top and on the bottom of a sphere are problematic for the smoothness and the accuracy of the solution. To overcome this problem C^1-continuity constrains where implemented to obtain more accurate results.

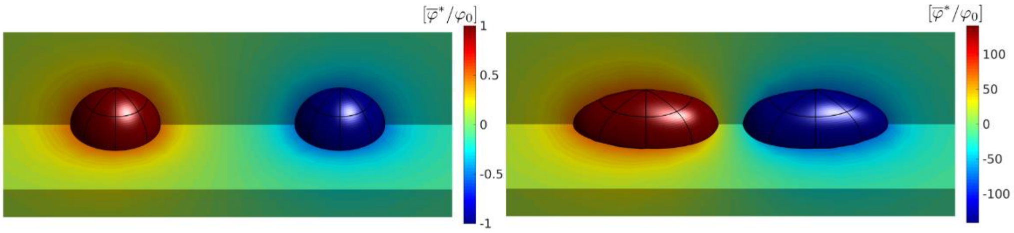

Figure 4. *Initial and deformed configuration for an electroelastic interaction.*

Summary and conclusion

The goals and the used numerical methods for TP B6 in the year 2018 are presented in this annual report. The capabilities of the developed and implemented formulations are demonstrated with some chosen examples. The progress brings TP B6 significantly closer to the overall goal, that is the development of a thermomechanical contact formulation between melt and component during the entire solidification progress.

In order to make further steps towards this goal, we will solve unsteady Stokes problems implementing the dual reciprocity BEM. To solve industrial applications, the efficiency of the BEM must be improved. A suitable method like the fast-multiple BEM or the adaptive cross approximation will be implemented. The efficient method for transient problems will be used to simulate problems that are relevant in other sub-projects of SFB 1120.

Präzision
aus Schmelze

Publication

HARMEL, M., RAJSKI, M.P., SAUER, R.A.: Desingularization in boundary element analysis of three-dimensional Stokes flow. *Proc. Appl. Math. Mech., 18:e201800479, 2018*